

Constraining the time variation of the coupling constants from cosmic microwave background: effect of Λ_{QCD}

Masahiro Nakashima ^{1,2}, Kazuhide Ichikawa ³, Ryo Nagata ², and Jun'ichi Yokoyama ^{2,4}

¹ *Department of Physics, Graduate School of Science,*

The University of Tokyo, Tokyo 113-0033, Japan

² *Research Center for the Early Universe (RESCEU),*

Graduate School of Science, The University of Tokyo, Tokyo 113-0033, Japan

³ *Department of Micro Engineering,*

Kyoto University, Kyoto 606-8501, Japan

⁴ *Institute for the Physics and Mathematics of the Universe (IPMU),*

The University of Tokyo, Kashiwa, Chiba, 277-8568, Japan

Abstract

We investigate constraints on the time variation of the fine structure constant between the recombination epoch and the present epoch, $\Delta\alpha/\alpha \equiv (\alpha_{\text{rec}} - \alpha_{\text{now}})/\alpha_{\text{now}}$, from cosmic microwave background (CMB) taking into account simultaneous variation of other physical constants, namely the electron mass m_e and the proton mass m_p . In other words, we consider the variation of Yukawa coupling and the QCD scale Λ_{QCD} in addition to the electromagnetic coupling. We clarify which parameters can be determined from CMB temperature anisotropy in terms of singular value decomposition. Assuming a relation among variations of coupling constants governed by a single scalar field (the dilaton), the 95% confidence level (C.L.) constraint on $\Delta\alpha/\alpha$ is found to be $-8.28 \times 10^{-3} < \Delta\alpha/\alpha < 1.81 \times 10^{-3}$, which is tighter than the one obtained by considering only the change of α and m_e . We also obtain the constraint on the time variation of the proton-to-electron mass ratio $\mu \equiv m_p/m_e$ to be $-0.52 < \Delta\mu/\mu < 0.17$ (95% C.L.) under the same assumption. Finally, we also implement a forecast for constraints from the PLANCK survey.

I. INTRODUCTION

Whether or not the physical constants are truly constant is not only a fundamental issue in physics but also an important probe of the theories of extra dimensions. Since the early studies of Dirac [1, 2], many theoretical models that could accommodate the variation of physical constants have been proposed [3, 4, 5, 6, 7]. Unification theories such as superstring theories predict the existence of an additional scalar field Φ called “dilaton” and the destabilization of this field [8, 9] might cause the variation of coupling constants. In more phenomenological contexts, dynamical scalar fields introduced to explain the recent discovery of the accelerated expansion may be non-minimally coupled to gauge fields in the Standard model, which may also lead to time varying coupling constants [10, 11, 12, 13, 14, 15]. Thus, investigating the variation of the coupling constants can be an important probe of these theories.

Not only these theoretical sides but also the observational analysis of various systems have been playing an important role in the investigation of the time variation of physical constants. Among them, the claim for the deviation of the past value of the fine structure constant α from the present one by using the high-redshift quasar absorption system [16, 17] had the great impact on this topic. Further analysis for the quasar absorption spectra gives the support for this positive result of a cosmological variation of α [18, 19, 20, 21], though some authors take objection to such variations [21, 22, 23].

Stimulated by these studies from both theoretical and observational sides, various constraints on the time variation of the fine structure constant α have been investigated from diverse observations. We briefly summarize those terrestrial and celestial limits on α as follows (see [6, 24] for a review). The atomic clocks constrain the current value of the temporal derivative of α as $\dot{\alpha}/\alpha = (-3.3 \pm 3.0) \times 10^{-16} \text{ yr}^{-1}$ [25, 26, 27, 28]. The measurement of the frequency ratio of aluminium and mercury single-ion optical clocks provides $\dot{\alpha}/\alpha = (-1.6 \pm 2.3) \times 10^{-17} \text{ yr}^{-1}$ [29]. From the analysis of Sm isotopes in the Oklo natural reactor in Gabon, we get two bounds on the variation of α as $\Delta\alpha/\alpha = -(0.8 \pm 1.0) \times 10^{-8}$ and $\Delta\alpha/\alpha = (0.88 \pm 0.07) \times 10^{-7}$ [30], which measures the value at the redshift $z \sim 0.1$ constraint. From the spectra of quasars, several limits have been obtained using various data sets: $\Delta\alpha/\alpha = (-0.57 \pm 0.11) \times 10^{-5}$ ($z \sim 0.2 - 4.2$) from the Keck/HIRES instrument [18], $\Delta\alpha/\alpha = (-0.64 \pm 0.36) \times 10^{-5}$ ($z \sim 0.4 - 2.3$) from the Ultraviolet

let and Visual Echelle Spectrograph (UVES) instrument [19, 20]. Big bang nucleosynthesis (BBN) provides constraints at very high redshifts ($z \sim 10^9 - 10^{10}$), for example, $-5.0 \times 10^{-2} < \Delta\alpha/\alpha < 1.0 \times 10^{-2}$ (95%C.L.) [31]. Finally, Cosmic Microwave Background (CMB) measures α at $z \sim 10^3$ and the limits from WMAP 1-year, 3-year and 5-year data read $-0.06 < \Delta\alpha/\alpha < 0.01$ [33], $-0.039 < \Delta\alpha/\alpha < 0.010$ [34] and $-0.028 < \Delta\alpha/\alpha < 0.026$ [35] respectively, all of which are at 95%C.L. Recently, Menegoni et al. [36] obtained more improved result $-0.013 < \Delta\alpha/\alpha < 0.015$ at 95%C.L. from WMAP 5-year data combined with ACBAR, QUAD and BICEP experiments data.

Along with α , several observational constraints have been obtained on another physical constant, the proton-to-electron mass ratio $\mu \equiv m_p/m_e$. As in α , atomic clocks very tightly constrain the current value of its temporal derivative as $\dot{\mu}/\mu = (-1.5 \pm 1.7) \times 10^{-15} \text{ yr}^{-1}$ [25]. Reinhold et al. [37] reports the non-vanishing variation of μ as $\Delta\mu/\mu = (2.4 \pm 0.6) \times 10^{-5}$ from a weighted sum of accurate H_2 spectral lines in two quasar systems (corresponding redshifts are $z = 2.59$ and $z = 3.02$). Based on the quasar absorption spectra of NH_3 , the limit is $\Delta\mu/\mu = (0.6 \pm 1.9) \times 10^{-6}$ at $z = 0.6847$ [38] and further detailed measurement by the same method tighten the bound as $\Delta\mu/\mu = (0.74 \pm 0.47) \times 10^{-6}$ [39].

In this paper, using WMAP 5-year data, we focus on the CMB constraint on α in the case that the multiple physical constants may vary with time simultaneously according to the expectation values of a single dilaton field Φ . Although similar analyses has been done for BBN [31, 40, 41, 42, 43, 44], quasar absorption system [45, 46] or the Oklo reactor [47], few attempts from CMB data have been made except for several preceding literatures [32, 48, 49], which treated simultaneous variations of only α and m_e . In addition to α and m_e , many theoretical models predict the time variation of the QCD scale Λ_{QCD} (which results from the variation of the strong coupling constant), so it is important to include the Λ_{QCD} effect in the analysis. We consider the effect of varying Λ_{QCD} as the change in the proton mass m_p which is assumed to be proportional to Λ_{QCD} ¹. Thus, we consider simultaneous variation of α , m_e and m_p . Since how the variation of Λ_{QCD} is related to that of α or m_e depends on the details of the unification model [50], we here adopt a specific

¹ When we consider the time variation of Λ_{QCD} , the neutron mass also changes. In this paper, we neglect the difference between the proton mass and the neutron mass, and when we refer to the time variation of the proton mass m_p , we implicitly include the time variation of the neutron mass assuming that the neutron mass changes in exactly the same way as the proton mass.

example of the relation among those physical constants following [31] or [40], which is based on a low energy effective theory of string theory. The newly incorporated change of Λ_{QCD} leads to a new and tighter constraint on the time variation of α . We also obtain a constraint on μ induced by the time dependence of Λ_{QCD} and m_e .

This paper is organized as follows. Section II provides how the variation of each physical constant affects the CMB power spectrum both qualitatively and quantitatively with particular attention to Λ_{QCD} . Section III briefly describes the model we adopt based on string theory which governs the time evolution of the three physical constants we consider, namely, the variation of the fine structure constant, the electron mass and the proton mass. In section IV, we provide the limits of those variations. Section V is devoted to conclusion. The details of the calculations of the degeneracy and parameter dependence are given in the appendix A. In appendix B, we present a forecast for constraints from the PLANCK experiment.

II. THE EFFECT OF VARIATIONS OF PHYSICAL CONSTANTS ON THE CMB

As is well known, a primary CMB photon is a relic signal from the last scattering surface about thirteen billion years ago, which means that the most important process for CMB photon is the recombination. It has been known that changing the value of the fine structure constant α and the electron mass m_e affects the CMB power spectrum mainly through the change of the epoch of recombination [51, 52, 53, 54]. The larger value of α or m_e at the recombination epoch causes the higher redshift of the last scattering surface because the hydrogen binding energy scales as $\alpha^2 m_e$. This results in three characteristic signatures in the angular power spectrum of the temperature anisotropy, namely, shift of the peaks to higher multipoles, increase of the height of the peaks due to the enhanced early integrated Sachs Wolfe effect, and decrease of the small-scale diffusion damping effect. To be more precise, however, the process of the recombination depend on α and m_e in a subtly different manner as will be explicitly shown in eqs.(6)-(9) in the argument of the four CMB-characteristic variables below.

The time variation of the proton mass m_p affects the CMB power spectrum mainly through the baryon density ρ_B . In the standard cosmology with no variation of physical constants, the baryon density scales as $\rho_{B,\text{rec}} = \rho_{B,0}(a_0/a_{\text{rec}})^3$, where a is the scale factor and

the subscripts “rec” and “0” denote values at the epoch of recombination and the present epoch, respectively. When we allow the time variation of m_p , there is an additional change as

$$\rho_{B,\text{rec}} = \rho_{B,0} \left(\frac{a_0}{a_{\text{rec}}} \right)^3 \frac{m_{p,\text{rec}}}{m_{p,0}} \equiv \rho_{B,0} \left(\frac{a_0}{a_{\text{rec}}} \right)^3 \left(1 + \frac{\Delta m_p}{m_p} \right), \quad (1)$$

where we define $\Delta m_p \equiv m_{p,\text{rec}} - m_{p,0}$. Therefore, the effect of changing $m_{p,\text{rec}}$ is very similar to changing $\rho_{B,0}$. One may even consider that $m_{p,\text{rec}}$ and $\rho_{B,0}$ are totally degenerate. However, there appears a subtle difference as regards the baryon number density n_B which affects the recombination process. This is because, contrary to the case of ρ_B , the scaling $n_{B,\text{rec}} = n_{B,0}(a_0/a_{\text{rec}})^3$ is not affected by the variation of m_p (note that $n_B \equiv \rho_B/m_p$). Thus, suppose one increases $\rho_{B,0}$ and decreases $m_{p,\text{rec}}$ while preserving the value of $\rho_{B,\text{rec}}$, $n_{B,\text{rec}}$ is increased. This is the reason why varying $\rho_{B,0}$ (as is done in the usual parameter search) and varying $m_{p,\text{rec}}$ are different and we can break the degeneracy between them in the parameter estimation although the degeneracy turns out to be rather strong.

With the careful attention to the difference between ρ_B and Δm_p noted in the last paragraph, we calculated the angular power spectra of the CMB temperature anisotropy, C_ℓ^{TT} , for several different values of Δm_p using the modified CAMB code [55]. The result is shown in FIG. 1, where we also draw the cross correlation of temperature and E-mode polarization, C_ℓ^{TE} , and the angular power spectrum of E-mode polarization, C_ℓ^{EE} , for completeness. Incorporating Δm_p into the numerical calculation, we assume m_p to change instantaneously at a specific time ($z = 100$). In other words, $m_{p,\text{rec}}$ was constant before $z = 100$ and after that the proton mass becomes the present value². This procedure is adopted throughout this paper. We consider a flat Λ CDM universe with a power-law adiabatic primordial fluctuation and take the fiducial standard cosmological parameter values as the WMAP 5-year marginalized mean values [56, 57];

$$(\Omega_B h^2, \Omega_{DM} h^2, h, \tau, n_s) = (0.0227, 0.1099, 0.72, 0.087, 0.963), \quad (2)$$

where Ω_B is the baryon energy density $\rho_{B,0}$ normalized by the critical density $\rho_{\text{crit},0}$, Ω_{DM} is the dark matter energy density $\rho_{DM,0}$ normalized by the critical density $\rho_{\text{crit},0}$, h is the current

² Here, only the mass difference Δm_p matters and it is not important how the proton mass has changed between the recombination epoch and the present epoch, because characteristic signatures such as acoustic oscillation or Silk damping are imprinted before and during the recombination process, which is also true on the effects that Δm_p induces.

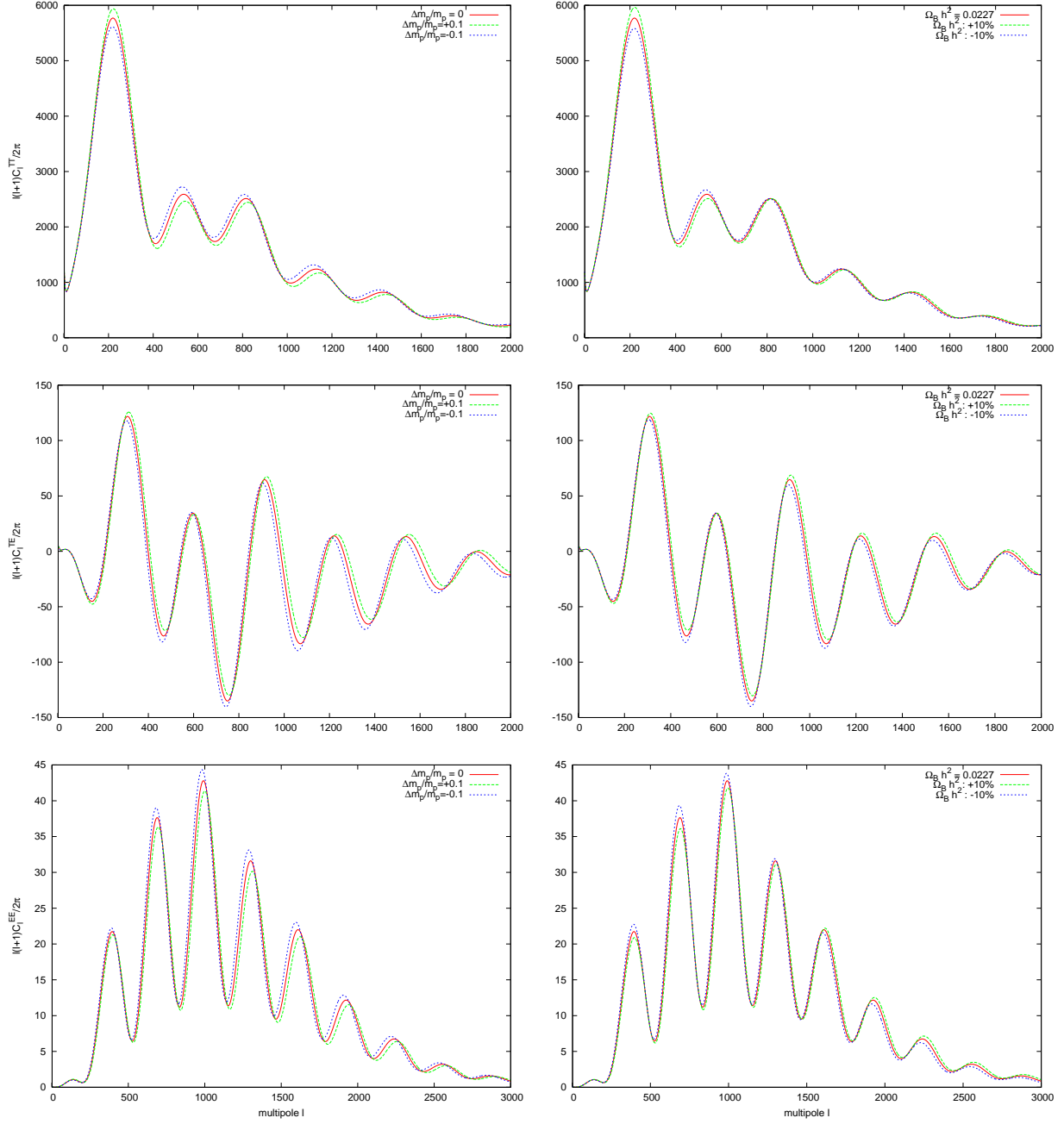


FIG. 1: The top left panel is the CMB temperature anisotropy spectra for no change of m_p (solid red curve), an increase of m_p by 10% (dashed green curve), a decrease of m_p by 10% (dotted blue curve). The middle left and bottom left panels are the TE and EE spectra respectively for three different values of $\Delta m_p/m_p$. For comparison, the corresponding spectra for three different values of $\Omega_B h^2$ are shown in the right hand side panels.

Hubble parameter, H_0 , in units of $100 \text{ km sec}^{-1} \text{ Mpc}^{-1}$, τ is the optical depth to reionization and n_s is the spectral index of the primordial curvature perturbation. As expected from the above argument, the effect of Δm_p is very similar to that of Ω_B . Due to the larger value of Δm_p , the height of the odd peaks increases compared with that of the even peaks, which is a very well-known result for Ω_B [58].

Now we discuss how the variations of the parameters change the shape of the CMB power spectrum quantitatively. We use four quantities which have been proposed to characterize the temperature spectrum well [59]: the position of the first peak ℓ_1 , the height of the first peak relative to the large angular-scale amplitude evaluated at $\ell = 10$,

$$H_1 \equiv \left(\frac{\Delta T_{\ell_1}}{\Delta T_{10}} \right)^2, \quad (3)$$

the ratio of the second peak ℓ_2 height to the first,

$$H_2 \equiv \left(\frac{\Delta T_{\ell_2}}{\Delta T_{\ell_1}} \right)^2, \quad (4)$$

the ratio of the third peak ℓ_3 height to the first,

$$H_3 \equiv \left(\frac{\Delta T_{\ell_3}}{\Delta T_{\ell_1}} \right)^2, \quad (5)$$

where $(\Delta T_\ell)^2 \equiv \ell(\ell+1)C_\ell^{\text{TT}}/2\pi$. We calculate the response of these four quantities when we vary the cosmological parameters. When we vary one parameter, the other parameters are fixed. The parameter set we consider is the standard flat- Λ CDM cosmological parameters plus varying physical constants: $\Omega_B h^2, \Omega_m h^2, h, \tau, n_s, \alpha_{\text{rec}}, m_{e,\text{rec}}, m_{p,\text{rec}}$, where Ω_m means $\Omega_B + \Omega_{DM}$. The fiducial values of the standard cosmological parameters are given in (2), and those of physical constants equal to the present values. Let us define $\omega_b = \Omega_B h^2, \omega_m = \Omega_m h^2$ and omit the subscript “rec” of physical constants for simplicity, then we have found after extensive numerical calculations that the four characteristic measures of the angular power spectrum have the following dependence on the cosmological parameters and the physical

constants.

$$\Delta\ell_1 = 15.1\frac{\Delta\omega_b}{\omega_b} - 24.0\frac{\Delta\omega_m}{\omega_m} - 45.3\frac{\Delta h}{h} + 35.6\frac{\Delta n_s}{n_s} - 317\frac{\Delta\alpha}{\alpha} + 142\frac{\Delta m_e}{m_e} + 19.1\frac{\Delta m_p}{m_p}, \quad (6)$$

$$\begin{aligned} \Delta H_1 = 2.84\frac{\Delta\omega_b}{\omega_b} - 2.80\frac{\Delta\omega_m}{\omega_m} - 2.02\frac{\Delta h}{h} + 18.3\frac{\Delta n_s}{n_s} - 0.64\frac{\Delta\tau}{\tau} \\ + 4.42\frac{\Delta\alpha}{\alpha} + 1.05\frac{\Delta m_e}{m_e} + 0.868\frac{\Delta m_p}{m_p}, \end{aligned} \quad (7)$$

$$\Delta H_2 = -0.29\frac{\Delta\omega_b}{\omega_b} - 0.020\frac{\Delta\omega_m}{\omega_m} + 0.53\frac{\Delta n_s}{n_s} + 0.75\frac{\Delta\alpha}{\alpha} + 0.20\frac{\Delta m_e}{m_e} - 0.351\frac{\Delta m_p}{m_p}, \quad (8)$$

$$\Delta H_3 = -0.18\frac{\Delta\omega_b}{\omega_b} + 0.19\frac{\Delta\omega_m}{\omega_m} + 0.33\frac{\Delta n_s}{n_s} + 0.42\frac{\Delta\alpha}{\alpha} - 0.008\frac{\Delta m_e}{m_e} - 0.246\frac{\Delta m_p}{m_p}, \quad (9)$$

where the values at the fiducial parameter values are $\ell_1 = 219.9$, $H_1 = 6.864$, $H_2 = 0.4482$ and $H_3 = 0.4342$, respectively. These formulae are extension of [32] to incorporate the change of m_p . The coefficients of the $\Delta\omega_b/\omega_b$ and $\Delta m_p/m_p$ are very similar, which agrees with our qualitative discussion. Although two parameters ω_b and m_p have a strong degeneracy, it is not exact and we can expect that this small but finite difference actually enables us to discriminate these two parameters and give some limit on $\Delta m_p/m_p$. More quantitative analysis of the degeneracy is performed in the appendix.

III. THE DILATON DEPENDENCE OF α , m_e AND m_p

In this section, we describe our setups for the dilaton dependence of the three physical constants and relation among them. This generally follows what is adopted in Refs. [31, 40]. As a concrete example, we consider the tree level low energy action of the heterotic string in the Einstein frame [40, 60, 61, 62, 63, 64, 65]. The action is

$$\begin{aligned} S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa^2} R - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \frac{1}{2} D_\mu \phi D^\mu \phi \right. \\ \left. - \Omega^{-2} V(\phi) - \bar{\psi} \gamma_\mu D^\mu \psi - \Omega^{-1} m_\psi \bar{\psi} \psi - \frac{\alpha'}{16\kappa^2} \Omega^2 F_{\mu\nu} F^{\mu\nu} \right), \end{aligned} \quad (10)$$

where $\kappa^2 = 8\pi G = 8\pi/M_{pl}^2$, where M_{pl} is the Planck scale, Φ is the dilaton field, ϕ is an arbitrary scalar field, ψ is an arbitrary fermion, D_μ is the gauge covariant derivative corresponding to gauge fields with field strength $F_{\mu\nu}$, and $F_{\mu\nu}$ is the gauge field with gauge group including $SU(3) \times SU(2) \times U(1)$. Ω is the conformal factor used to transfer from the string frame and defined as

$$\Omega = e^{-\kappa\Phi/\sqrt{2}}. \quad (11)$$

Comparing the gauge field strength term $-(\alpha'/16\kappa^2)\Omega^2 F_{\mu\nu}F^{\mu\nu}$ of the action (10) with the definition of the Lagrangian density $-(1/4g^2)F_{\mu\nu}F^{\mu\nu}$ where g is the unified coupling constant, we get

$$\frac{1}{g^2(M_{pl})} = \frac{\alpha' e^{-\sqrt{2}\kappa\Phi}}{4\kappa^2} \equiv \frac{\alpha' S}{4\kappa}. \quad (12)$$

The gauge coupling constants at low energy scale are calculated using renormalization group equations. As for the fine structure constant α , we can check that α does not run practically, so α at low energy (or CMB energy scale which is equal to the energy scale at which recombination took place) is determined in good approximation as

$$\alpha_{\text{rec}} \approx \alpha(M_{pl}) = \frac{g(M_{pl})^2}{4\pi} = \frac{\kappa^2}{\pi\alpha'} e^{\sqrt{2}\kappa\Phi}. \quad (13)$$

From this expression, our variable $\Delta\alpha/\alpha$ becomes

$$\frac{\Delta\alpha}{\alpha} = \frac{\alpha_{\text{rec}} - \alpha_0}{\alpha_0} = e^{\sqrt{2}\kappa\Delta\Phi} - 1 \quad (14)$$

where $\Delta\Phi = \Phi_{\text{rec}} - \Phi_0$.

From the solution of the one-loop renormalization group equation for the SU(3) coupling constant g_3^3 ,

$$\frac{\kappa^2}{\pi\alpha'} e^{\sqrt{2}\kappa\Phi} = \frac{g_3(M_{pl})^2}{4\pi} \simeq \frac{12\pi}{27 \log(M_{pl}^2/\Lambda_{\text{QCD}}^2)}. \quad (15)$$

Using this relation and the fact that $g(M_{pl})^2 = 0.1$ and that the proton mass m_p is proportional to the QCD energy scale Λ_{QCD} , $\Delta m_p/m_p$ is also expressed through $\Delta\alpha/\alpha$ as

$$\frac{\Delta m_p}{m_p} \equiv \frac{m_{p,\text{rec}} - m_{p,\text{now}}}{m_{p,\text{now}}} = \frac{\Delta\Lambda_{\text{QCD}}}{\Lambda_{\text{QCD}}} = \exp\left(\frac{80\pi^2}{9} \frac{\frac{\Delta\alpha}{\alpha}}{1 + \frac{\Delta\alpha}{\alpha}}\right) - 1. \quad (16)$$

This relation is peculiar in that a small increase in $\Delta\alpha/\alpha$ results in a large increase in $\Delta m_p/m_p$, which has a great impact on the parameter estimation.

Finally, we can read the electron mass term from the action as $e^{\kappa\Phi\sqrt{2}}m_\psi\bar{\psi}\psi$. Assuming that the Higgs vacuum expectation value $\langle H \rangle = \text{const.}$, the time variation of m_e is expressed as

$$\frac{\Delta m_e}{m_e} = \frac{m_{e,\text{rec}} - m_{e,0}}{m_{e,0}} = e^{\kappa\Delta\Phi/\sqrt{2}} = \sqrt{1 + \frac{\Delta\alpha}{\alpha}} - 1. \quad (17)$$

³ The integration constant is determined by $g_3(\Lambda_{\text{QCD}}) = \infty$

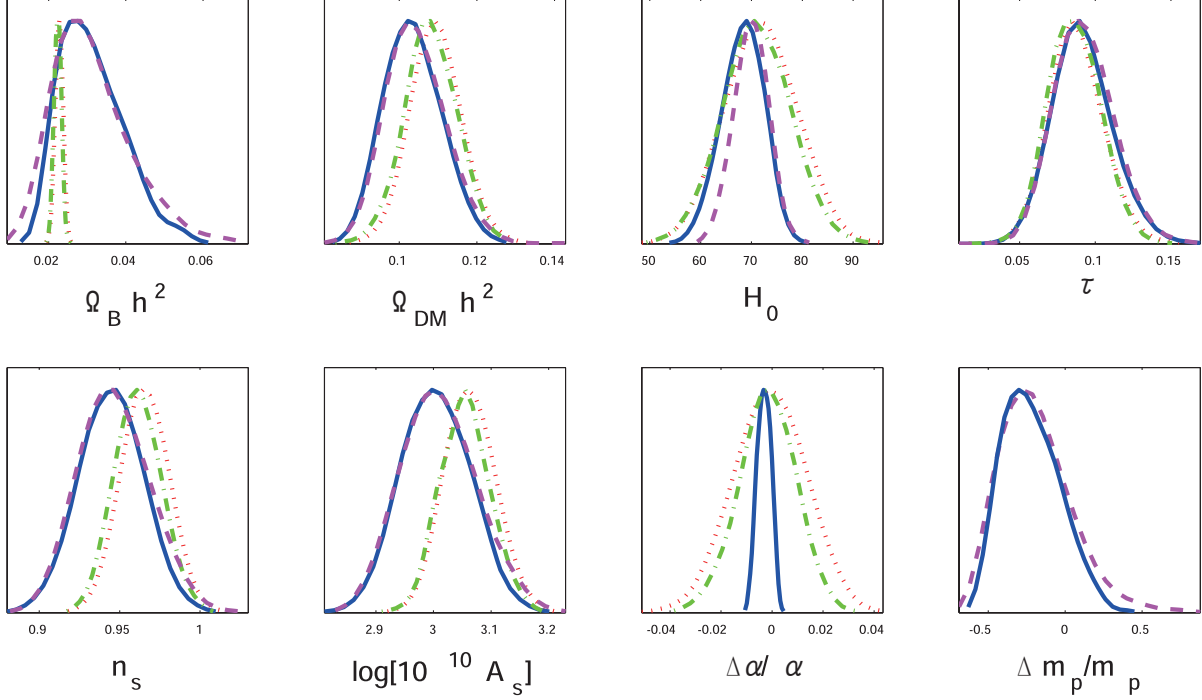


FIG. 2: One-dimensional marginalized posterior distributions for four models: varying all three constants α, m_e and m_p assuming the relation (16) and (17) (solid blue curves), varying only two constants α and m_e assuming the relation (17) (dot-dashed green curves), varying only one constant α (dotted red curves) and varying only one constant m_p (dashed magenta curves).

IV. CONSTRAINTS ON THE VARIATION OF THE COUPLING CONSTANTS

Based on the model described in the previous section, we constrain the variation of α using three kinds of CMB anisotropy spectra, C_ℓ^{TT} , C_ℓ^{EE} , and C_ℓ^{TE} of the five-year WMAP data [56, 57, 66]. Theoretical anisotropy spectra are calculated by the CAMB code [55, 67] modified to include the varying physical constants as in Sec. II. We performed the parameter estimation using Markov-Chain Monte-Carlo (MCMC) techniques implemented in the public CosmoMC code [68, 69].

We have run the CosmoMC code on eight Markov chains in each case. To check the convergence, we used the “variance of chain means”/“mean of chain variances” R statistic and adopted the condition $R - 1 < 0.03$. In all the analysis below, we have also incorporated the result of Hubble Key Project of the Hubble Space Telescope (HST) on the Hubble parameter H_0 , which means that we have imposed Gaussian prior of

TABLE I: MCMC results on the mean values and 68% confidence intervals of cosmological parameters for four versions: varying α only, varying α and m_e , varying α, m_e and m_p , varying m_p only.

	varying α only	varying α and m_e	varying α, m_e and m_p	varying m_p only
$100\Omega_B h^2$	$2.27^{+0.10}_{-0.10}$	$2.24^{+0.09}_{-0.09}$	$3.13^{+0.85}_{-0.83}$	$3.13^{+0.97}_{-0.95}$
$\Omega_{DM} h^2$	$0.109^{+0.006}_{-0.006}$	$0.108^{+0.007}_{-0.006}$	$0.103^{+0.008}_{-0.007}$	$0.104^{+0.007}_{-0.007}$
τ	$0.0877^{+0.007}_{-0.008}$	$0.0864^{+0.008}_{-0.009}$	$0.0916^{+0.007}_{-0.010}$	$0.0928^{+0.008}_{-0.010}$
n_s	$0.966^{+0.014}_{-0.015}$	$0.961^{+0.014}_{-0.015}$	$0.945^{+0.020}_{-0.020}$	$0.947^{+0.022}_{-0.022}$
$\log(10^{10} A_s)$	$3.06^{+0.05}_{-0.04}$	$3.06^{+0.04}_{-0.04}$	$3.00^{+0.06}_{-0.06}$	$3.01^{+0.06}_{-0.07}$
H_0	$71.9^{+7.0}_{-7.1}$	$71.0^{+6.4}_{-6.4}$	$68.2^{+4.2}_{-4.2}$	$69.9^{+3.3}_{-3.4}$
$\Delta\alpha/\alpha$	$-0.00089^{+0.0131}_{-0.0148}$	$-0.0019^{+0.011}_{-0.011}$	$-0.0031^{+0.0028}_{-0.0029}$	—
$\Delta m_p/m_p$	—	—	$-0.22^{+0.19}_{-0.19}$	$-0.19^{+0.22}_{-0.23}$

$H_0 = 72 \pm 8 \text{ km sec}^{-1} \text{ Mpc}^{-1}$ [70]. In our analysis, we derive the confidence limits for standard parameters ($\Omega_B h^2, \Omega_{DM} h^2, H_0, n_s, A_s, \tau$) plus some varying physical constants.

Figures 2 show 1D-marginalized posterior distributions of the eight parameters for the case not only α but also m_e and m_p are changed simultaneously according to the model described in the previous section. This is the main result of this paper and for comparison with the literature we have also plotted the distributions for two other cases: the case of varying only α [35] and the case of varying both α and m_e assuming the relation (17) which has already been analyzed in [49]. In TABLE I, we also present the mean values and the 68% confidence intervals of the cosmological parameters for these three cases. We can conclude from these results that the inclusion of the variation of m_p results in a tighter limit on the variation of α , which is now

$$-8.28 \times 10^{-3} < \frac{\Delta\alpha}{\alpha} < 1.81 \times 10^{-3}, \quad (18)$$

compared with the limit for the two other cases: $-0.024 < \Delta\alpha/\alpha < 0.019$ (for varying α, m_e) and $-0.028 < \Delta\alpha/\alpha < 0.026$ (for varying only α), all of which are at 95% C.L.

In FIG. 2 and TABLE I, we can also see that, as a cost of tightening of the constraint on α , the limit on $\Omega_B h^2$, which is one of the most tightly constrained parameter from CMB

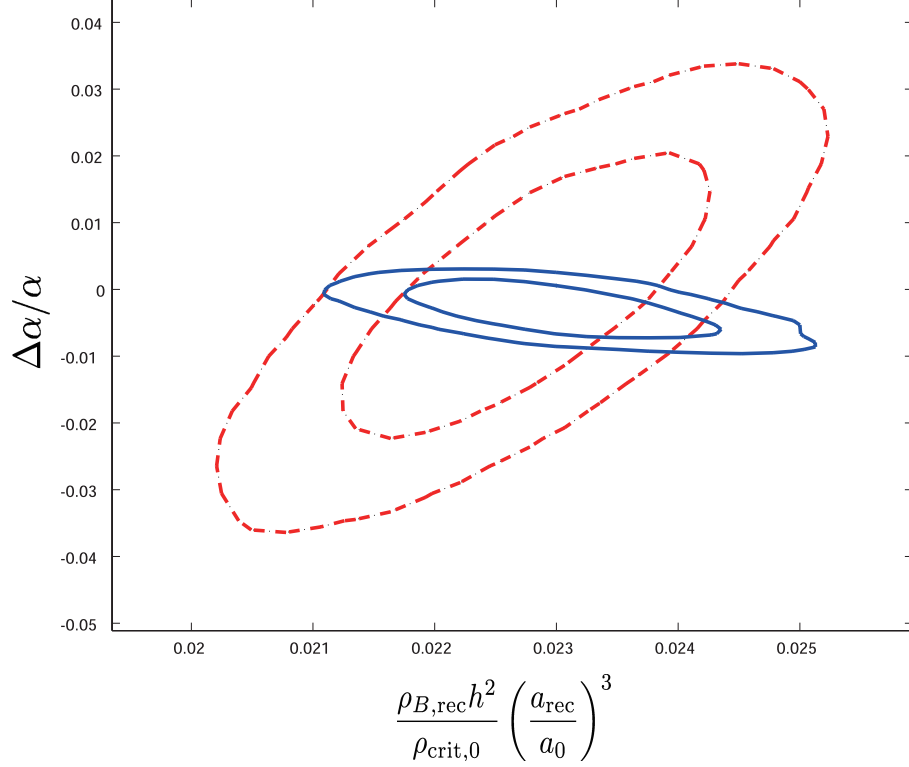


FIG. 3: Two-dimensional marginalized posterior distributions of $\rho_{B,\text{rec}}h^2/\rho_{\text{crit},0}$ and $\Delta\alpha/\alpha$ for two models: varying all three constants α, m_e and m_p assuming the relation (16) and (17) (solid blue contours) and varying only one constant α (dot-dashed red contours). The plotted $\rho_{B,\text{rec}}h^2/\rho_{\text{crit},0}$ (horizontal axis) is actually $(\rho_{B,\text{rec}}h^2/\rho_{\text{crit},0})(a_{\text{rec}}/a_0)^3$, which corresponds to $(\rho_{B,0}h^2/\rho_{\text{crit},0})(1 + \Delta m_p/m_p)$ for the former case and $\rho_{B,0}h^2/\rho_{\text{crit},0}$ for the latter case. In each pair of contours, the inner one shows 68%C.L. and the outer one shows 95%C.L.

data in ordinary analysis, become very loose because of the degeneracy between Ω_B and $\Delta m_p/m_p$. This result is reasonable from the argument in Sec. II. On the other hand, since CMB is sensitive to the values of cosmological parameters at the recombination epoch, the limit on $\Omega_{B,\text{rec}}$ in the present scenario is expected to be of the same order as that on $\Omega_{B,\text{rec}}$ in the case without m_p variation. We can confirm it from FIG. 3, which shows 2D-marginalized posterior distributions of $\Delta\alpha/\alpha - \rho_{B,\text{rec}}h^2/\rho_{\text{crit},0}$ for two cases, one with both m_p and m_e variation and the other with no variation in m_p and m_e .

One of the reasons why we could obtain tighter constraint than the case without varying m_p is thought to be the specific relation (16) in our model. To ensure that statement, we have also implemented MCMC calculations in the case that only m_p is varied and the other

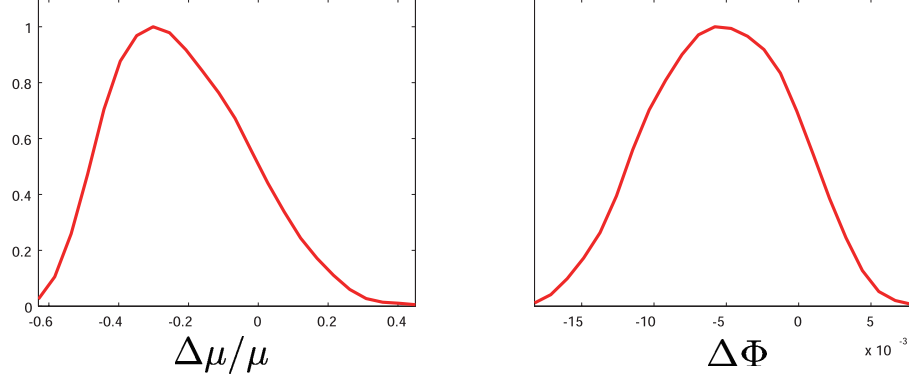


FIG. 4: One-dimensional marginalized posterior distributions for two parameters: $\Delta\mu/\mu$ and $\Delta\Phi$

two physical constants remain constant, which means that we do not impose any model in this case. The result is also shown in FIG. 2 as 1D-marginalized posterior distributions. Two marginalized curves in the panel showing $\Delta m_p/m_p$ are very similar and those limits at 95%C.L. are $-0.54 < \Delta m_p/m_p < 0.36$ (for varying m_p only) and $-0.51 < \Delta m_p/m_p < 0.17$ (for varying α, m_e and m_p), which implies that the dominant factor in constraining the variation of physical constants under the relation (17) and (16) is $\Delta m_p/m_p$ and it is the specific relation between $\Delta\alpha/\alpha$ and $\Delta m_p/m_p$ that determines the order of magnitude of the resultant $\Delta\alpha/\alpha$ limit in the case these parameters are varied simultaneously through the dilaton's motion.

Finally, in our model, we can calculate the limit on not only $\Delta\alpha/\alpha$ but also $\Delta\Phi$ and $\Delta\mu/\mu$ where $\mu \equiv m_p/m_e$. The results are shown in FIG. 4. $\Delta\Phi$ is the dilaton field variation and using the relation (14) with $\kappa = \sqrt{8\pi G} = (2.43 \times 10^{18}[\text{GeV}])^{-1}$, we obtain at 95%C.L.

$$-1.4 \times 10^{16}[\text{GeV}] < \Delta\Phi < 3.1 \times 10^{15}[\text{GeV}]. \quad (19)$$

On the other hand, the constraint on $\Delta\mu/\mu$ is, at 95%C.L.,

$$-0.52 < \frac{\Delta\mu}{\mu} < 0.17. \quad (20)$$

Note that, although it is not so stringent compared with those obtained from other experiments [37], constraining this parameter in the recombination epoch is meaningful because in the previous works the variation of m_p has never been considered. We also expect that future CMB measurement such as PLANCK will give more stringent limits of these parameters (see Appendix B).

V. CONCLUSION

In summary, we have pointed out that the time variation of Λ_{QCD} can affect the CMB through the variation in the proton mass. We have shown that there is a notable effect when we impose the widely adopted assumptions of unification models found in the literature [40]. From the WMAP 5-year data, by MCMC analysis using the CosmoMC code, we have obtained the new and stringent constraint on the time variation of the fine structure constant as $-8.28 \times 10^{-3} < \Delta\alpha/\alpha < 1.81 \times 10^{-3}$. We have also verified that, in getting the above result, the newly considered effect of $\Delta m_p/m_p$ is very important in that the degeneracy between $\Delta m_p/m_p$ and Ω_B is tremendously strong and that the small variation of $\Delta\alpha/\alpha$ leads to the large variation of $\Delta m_p/m_p$. Furthermore, by including the varying m_p , we have obtained a limit on the variation of proton-electron mass ratio μ as $-0.52 < \Delta\mu/\mu < 0.17$.

CMB observation will be much more precise to give a lot of information around the damping tail of the power spectrum in near future. The difference in the CMB power spectra between $\Delta m_p/m_p$ and Ω_B appears in the small scale diffusion damping because the damping scale is determined by the baryon number density n_B , probing the time variation of coupling constants including Λ_{QCD} by CMB will be even more promising. This is discussed more quantitatively in appendix A, and supported by a forecast for future CMB survey in appendix B.

Acknowledgements

We would like to thank Asantha Cooray for useful comments and Toyokazu Sekiguchi for useful help. M.N. is supported by JSPS through research fellowships. This work was partially supported by JSPS Grant-in-Aid for Scientific Research No. 19340054 and Global COE Program "The Physical Sciences Frontier," MEXT, Japan.

Appendix A

In this appendix, we discuss the degeneracies between cosmological parameters using the linear algebra. In section II, we have introduced four variables ℓ_1, H_1, H_2, H_3 and calculated how they depend on the cosmological parameters and the physical constants

$(\omega_b, \omega_m, h, \tau, n_s, \alpha_{\text{rec}}, m_{e,\text{rec}}, m_{p,\text{rec}})$, the results of which are shown in (6)-(9). These are rewritten in the matrix form as follows.

$$\begin{pmatrix} \frac{\Delta \ell_1}{\ell_1} \\ \frac{\Delta H_1}{H_1} \\ \frac{\Delta H_2}{H_2} \\ \frac{\Delta H_3}{H_3} \end{pmatrix} = \begin{pmatrix} 0.069 & -0.109 & -0.206 & 0.162 & 0 & -1.442 & 0.646 & 0.087 \\ 0.414 & -0.408 & -0.294 & 2.666 & -0.093 & 0.644 & 0.153 & 0.127 \\ -0.647 & -0.045 & 0 & 1.183 & 0 & 1.673 & 0.446 & -0.783 \\ -0.415 & 0.438 & 0 & 0.760 & 0 & 0.967 & -0.018 & -0.567 \end{pmatrix} \begin{pmatrix} \frac{\Delta \omega_b}{\omega_b} \\ \frac{\Delta \omega_m}{\omega_m} \\ \frac{\Delta h}{h} \\ \frac{\Delta n_s}{n_s} \\ \frac{\Delta \tau}{\tau} \\ \frac{\Delta \alpha}{\alpha} \\ \frac{\Delta m_e}{m_e} \\ \frac{\Delta m_p}{m_p} \end{pmatrix}, \quad (21)$$

where we have divided the both sides of each equation by the fiducial values $\ell_1 = 219.9$, $H_1 = 6.864$, $H_2 = 0.4482$, $H_3 = 0.4342$, respectively. The 4×8 matrix connects the cosmological parameters and the CMB variables. Such a rectangular $n \times m$ matrix, which we call A , can be decomposed in general as

$$A = U \Sigma V^T \quad (U : n \times n, \Sigma : n \times m, V : m \times m), \quad (22)$$

which is known as the singular value decomposition. If $n < m$ and $\text{rank} A = n$, then the form of Σ is like

$$\Sigma = \left(\begin{array}{ccc|c} \sigma_1 & & & 0 \\ & \ddots & & \\ & & \sigma_n & \\ \hline & & & 0 \end{array} \right), \quad (23)$$

where σ_i is the non-zero singular value of the matrix A and the matrices U and V are orthonormal ones. Applying the singular value decomposition to the 4×8 matrix in (21)

and using the notation $D = U\Sigma$, we obtain

$$D = \begin{pmatrix} -0.672 & -1.227 & -0.794 & 0.033 & 0 & 0 & 0 & 0 \\ 2.463 & -1.350 & 0.322 & 0.008 & 0 & 0 & 0 & 0 \\ 2.1145 & 0.815 & -0.488 & -0.238 & 0 & 0 & 0 & 0 \\ 1.268 & 0.614 & -0.232 & 0.398 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (24)$$

$$V^T = \begin{pmatrix} -0.073 & -0.037 & -0.046 & 0.788 & -0.018 & 0.581 & 0.069 & -0.168 \\ -0.326 & 0.209 & 0.149 & -0.542 & 0.029 & 0.654 & -0.148 & -0.289 \\ 0.478 & -0.121 & 0.067 & -0.024 & -0.029 & 0.304 & -0.660 & 0.473 \\ -0.028 & 0.825 & -0.042 & 0.222 & -0.003 & -0.253 & -0.419 & -0.166 \\ 0.019 & -0.013 & 0.027 & 0.0350 & 0.998 & -0.006 & -0.020 & 0.013 \\ 0.702 & 0.331 & -0.318 & -0.169 & 0.018 & 0.230 & 0.450 & -0.106 \\ 0.0670 & 0.285 & 0.817 & 0.074 & -0.019 & 0.049 & 0.347 & 0.344 \\ -0.402 & 0.261 & -0.447 & -0.028 & 0.019 & 0.153 & 0.196 & 0.712 \end{pmatrix} \quad (25)$$

Now, (21) becomes

$$\begin{pmatrix} \frac{\Delta \ell_1}{\ell_1} \\ \frac{\Delta H_1}{H_1} \\ \frac{\Delta H_2}{H_2} \\ \frac{\Delta H_3}{H_3} \end{pmatrix} = DV^T \begin{pmatrix} \frac{\Delta \omega_b}{\omega_b} \\ \frac{\Delta \omega_m}{\omega_m} \\ \frac{\Delta h}{h} \\ \frac{\Delta n_s}{n_s} \\ \frac{\Delta \tau}{\tau} \\ \frac{\Delta m_p}{m_p} \end{pmatrix} \approx D \begin{pmatrix} 0.788 \frac{\Delta n_s}{n_s} + 0.581 \frac{\Delta \alpha}{\alpha} \\ -0.542 \frac{\Delta n_s}{n_s} + 0.654 \frac{\Delta \alpha}{\alpha} \\ 0.478 \frac{\Delta \omega_b}{\omega_b} - 0.660 \frac{\Delta m_e}{m_e} + 0.473 \frac{\Delta m_p}{m_p} \\ 0.825 \frac{\Delta \omega_m}{\omega_m} - 0.419 \frac{\Delta m_e}{m_e} \\ 0.999 \frac{\Delta \tau}{\tau} \\ -0.702 \frac{\Delta \omega_b}{\omega_b} + 0.451 \frac{\Delta m_e}{m_e} \\ 0.817 \frac{\Delta h}{h} \\ -0.402 \frac{\Delta \omega_b}{\omega_b} - 0.447 \frac{\Delta h}{h} + 0.712 \frac{\Delta m_p}{m_p} \end{pmatrix} \quad (26)$$

At the last transformation, we have neglected the terms whose coefficients are small and irrelevant. Remembering the explicit form of the matrix D that the last four columns are zero, the last four components in the column vector represent the four degeneracy directions, $0.999\Delta\tau/\tau$, $-0.702\Delta\omega_b/\omega_b + 0.451\Delta m_e/m_e$, $0.817\Delta h/h$, $-0.402\Delta\omega_b/\omega_b - 0.447\Delta h/h + 0.712\Delta m_p/m_p$.

In the above case, where all the variations of physical constants are included in the analysis, the final results are complicated and it is difficult to understand the degeneracies clearly. Therefore, we consider simpler cases in order now.

First, we treat only standard cosmological parameters, so (21) is reduced to

$$\begin{pmatrix} \frac{\Delta \ell_1}{\ell_1} \\ \frac{\Delta H_1}{H_1} \\ \frac{\Delta H_2}{H_2} \\ \frac{\Delta H_3}{H_3} \end{pmatrix} = \begin{pmatrix} 0.069 & -0.109 & -0.206 & 0.162 & 0 \\ 0.414 & -0.408 & -0.294 & 2.666 & -0.093 \\ -0.647 & -0.045 & 0 & 1.183 & 0 \\ -0.415 & 0.438 & 0 & 0.760 & 0 \end{pmatrix} \begin{pmatrix} \frac{\Delta \omega_b}{\omega_b} \\ \frac{\Delta \omega_m}{\omega_m} \\ \frac{\Delta h}{h} \\ \frac{\Delta n_s}{n_s} \\ \frac{\Delta \tau}{\tau} \end{pmatrix}. \quad (27)$$

The singular value decomposition of the 4×5 matrix gives

$$D = \begin{pmatrix} -0.190 & 0.131 & -0.052 & -0.172 & 0 \\ -2.713 & 0.420 & 0.039 & 0.017 & 0 \\ -1.171 & -0.626 & -0.239 & 0.006 & 0 \\ -0.708 & -0.609 & 0.261 & -0.031 & 0 \end{pmatrix}, V^T = \begin{pmatrix} -0.009 & 0.094 & 0.090 & -0.991 & 0.027 \\ 0.879 & -0.443 & -0.157 & -0.066 & -0.041 \\ 0.452 & 0.888 & -0.005 & 0.0787 & -0.028 \\ 0.141 & -0.075 & 0.982 & 0.080 & -0.053 \\ 0.056 & 0 & 0.043 & 0.031 & 0.997 \end{pmatrix}, \quad (28)$$

and we get the approximated relation

$$\begin{pmatrix} \frac{\Delta \ell_1}{\ell_1} \\ \frac{\Delta H_1}{H_1} \\ \frac{\Delta H_2}{H_2} \\ \frac{\Delta H_3}{H_3} \end{pmatrix} = D V^T \begin{pmatrix} \frac{\Delta \omega_b}{\omega_b} \\ \frac{\Delta \omega_m}{\omega_m} \\ \frac{\Delta h}{h} \\ \frac{\Delta n_s}{n_s} \\ \frac{\Delta \tau}{\tau} \end{pmatrix} \approx D \begin{pmatrix} -0.991 \frac{\Delta n_s}{n_s} \\ 0.879 \frac{\Delta \omega_b}{\omega_b} - 0.443 \frac{\Delta \omega_m}{\omega_m} \\ 0.452 \frac{\Delta \omega_b}{\omega_b} + 0.888 \frac{\Delta \omega_m}{\omega_m} \\ 0.983 \frac{\Delta h}{h} \\ 0.997 \frac{\Delta \tau}{\tau} \end{pmatrix}. \quad (29)$$

This tells us that the four standard cosmological parameters, n_s, ω_b, ω_m and h can be determined by observing the four variables ℓ_1, H_1, H_2 and H_3 , however the optical depth τ cannot be limited sufficiently, which is the direct consequence of the fact that we cannot tightly constrain τ only through the temperature anisotropy spectrum.

In the next case, we add one physical constant $\Delta\alpha/\alpha$ to the parameter vector;

$$\begin{pmatrix} \frac{\Delta \ell_1}{\ell_1} \\ \frac{\Delta H_1}{H_1} \\ \frac{\Delta H_2}{H_2} \\ \frac{\Delta H_3}{H_3} \end{pmatrix} = \begin{pmatrix} 0.069 & -0.109 & -0.206 & 0.162 & 0 & -1.442 \\ 0.414 & -0.408 & -0.294 & 2.666 & -0.093 & 0.644 \\ -0.647 & -0.045 & 0 & 1.183 & 0 & 1.673 \\ -0.415 & 0.438 & 0 & 0.760 & 0 & 0.967 \end{pmatrix} \begin{pmatrix} \frac{\Delta \omega_b}{\omega_b} \\ \frac{\Delta \omega_m}{\omega_m} \\ \frac{\Delta h}{h} \\ \frac{\Delta n_s}{n_s} \\ \frac{\Delta \tau}{\tau} \\ \frac{\Delta \alpha}{\alpha} \end{pmatrix}. \quad (30)$$

From the singular value decomposition and the relevant approximation, we obtain

$$D = \begin{pmatrix} -0.694 & -1.219 & -0.436 & -0.094 & 0 & 0 \\ 2.540 & -1.2123 & 0.172 & 0.045 & 0 & 0 \\ 1.970 & 0.810 & -0.162 & -0.239 & 0 & 0 \\ 1.182 & 0.541 & -0.354 & 0.246 & 0 & 0 \end{pmatrix} \quad (31)$$

$$V^T = \begin{pmatrix} -0.062 & -0.044 & -0.049 & 0.810 & -0.019 & 0.579 \\ -0.342 & 0.212 & 0.156 & -0.528 & 0.029 & 0.731 \\ 0.789 & -0.459 & 0.106 & -0.200 & -0.043 & 0.337 \\ 0.500 & 0.861 & 0.048 & 0.069 & -0.032 & 0.025 \\ 0.056 & 0 & 0.051 & 0.032 & 0.997 & -0.001 \\ 0.062 & 0.029 & -0.978 & -0.141 & 0.052 & 0.125 \end{pmatrix} \quad (32)$$

and

$$\begin{pmatrix} \frac{\Delta \ell_1}{\ell_1} \\ \frac{\Delta H_1}{H_1} \\ \frac{\Delta H_2}{H_2} \\ \frac{\Delta H_3}{H_3} \end{pmatrix} = DV^T \begin{pmatrix} \frac{\Delta \omega_b}{\omega_b} \\ \frac{\Delta \omega_m}{\omega_m} \\ \frac{\Delta h}{h} \\ \frac{\Delta n_s}{n_s} \\ \frac{\Delta \tau}{\tau} \\ \frac{\Delta \alpha}{\alpha} \end{pmatrix} \approx D \begin{pmatrix} 0.810 \frac{\Delta n_s}{n_s} + 0.579 \frac{\Delta \alpha}{\alpha} \\ -0.342 \frac{\Delta \omega_b}{\omega_b} - 0.528 \frac{\Delta n_s}{n_s} + 0.731 \frac{\Delta \alpha}{\alpha} \\ 0.789 \frac{\Delta \omega_b}{\omega_b} - 0.459 \frac{\Delta \omega_m}{\omega_m} + 0.337 \frac{\Delta \alpha}{\alpha} \\ 0.500 \frac{\Delta \omega_b}{\omega_b} + 0.861 \frac{\Delta \omega_m}{\omega_m} \\ 0.997 \frac{\Delta \tau}{\tau} \\ -0.978 \frac{\Delta h}{h} \end{pmatrix}. \quad (33)$$

The last equation (33) makes a contrast with (29). The newly included parameter $\Delta\alpha/\alpha$ is now determined together with n_s , ω_b and ω_m , on the other hand the Hubble parameter h becomes the unlimited parameter like τ . This is exactly what is expected because in the previous paper [35] we have made sure that, in constraining $\Delta\alpha/\alpha$, the HST prior on the Hubble parameter is very useful, which implied that including $\Delta\alpha/\alpha$ in the analysis weakens the constraint on h .

As the third case, which we are most interested in, we add the new parameter $\Delta m_p/m_p$;

$$\begin{pmatrix} \frac{\Delta \ell_1}{\ell_1} \\ \frac{\Delta H_1}{H_1} \\ \frac{\Delta H_2}{H_2} \\ \frac{\Delta H_3}{H_3} \end{pmatrix} = \begin{pmatrix} 0.069 & -0.109 & -0.206 & 0.162 & 0 & 0.087 \\ 0.414 & -0.408 & -0.294 & 2.666 & -0.093 & 0.127 \\ -0.647 & -0.045 & 0 & 1.183 & 0 & -0.783 \\ -0.415 & 0.438 & 0 & 0.760 & 0 & -0.567 \end{pmatrix} \begin{pmatrix} \frac{\Delta \omega_b}{\omega_b} \\ \frac{\Delta \omega_m}{\omega_m} \\ \frac{\Delta h}{h} \\ \frac{\Delta n_s}{n_s} \\ \frac{\Delta \tau}{\tau} \\ \frac{\Delta m_p}{m_p} \end{pmatrix}. \quad (34)$$

Similar decomposition and approximation lead to

$$D = \begin{pmatrix} -0.177 & 0.170 & -0.056 & 0.172 & 0 & 0 \\ -2.676 & 0.629 & 0.0305 & -0.017 & 0 & 0 \\ -1.260 & -0.890 & -0.230 & -0.006 & 0 & 0 \\ -0.776 & -0.763 & 0.281 & 0.031 & 0 & 0 \end{pmatrix} \quad (35)$$

$$V^T = \begin{pmatrix} 0.002 & 0.088 & 0.088 & -0.985 & 0.027 & 0.114 \\ 0.647 & -0.316 & -0.122 & 0.040 & -0.033 & 0.681 \\ 0.301 & 0.936 & 0.0190 & 0.102 & -0.0210 & 0.145 \\ -0.139 & 0.075 & -0.983 & -0.080 & 0.053 & -0.002 \\ 0.0380 & 0.003 & 0.046 & 0.033 & 0.997 & 0.019 \\ -0.685 & 0.098 & 0.095 & 0.098 & 0.004 & 0.709 \end{pmatrix} \quad (36)$$

and

$$\begin{pmatrix} \frac{\Delta \ell_1}{\ell_1} \\ \frac{\Delta H_1}{H_1} \\ \frac{\Delta H_2}{H_2} \\ \frac{\Delta H_3}{H_3} \end{pmatrix} = DV^T \begin{pmatrix} \frac{\Delta \omega_b}{\omega_b} \\ \frac{\Delta \omega_m}{\omega_m} \\ \frac{\Delta h}{h} \\ \frac{\Delta n_s}{n_s} \\ \frac{\Delta \tau}{\tau} \\ \frac{\Delta m_p}{m_p} \end{pmatrix} \approx D \begin{pmatrix} -0.985 \frac{\Delta n_s}{n_s} \\ -0.647 \frac{\Delta \omega_b}{\omega_b} + 0.681 \frac{\Delta m_p}{m_p} \\ 0.936 \frac{\Delta \omega_m}{\omega_m} \\ -0.983 \frac{\Delta h}{h} \\ 0.997 \frac{\Delta \tau}{\tau} \\ -0.685 \frac{\Delta \omega_b}{\omega_b} + 0.709 \frac{\Delta m_p}{m_p} \end{pmatrix}. \quad (37)$$

It is found that there are two degeneracy directions and one of them is $-0.685\Delta\omega_b/\omega_b + 0.709\Delta m_p/m_p$. This means that ω_b and m_p are strongly degenerate, which could be naively expected from just comparing the coefficients of the two parameters $\Delta\omega_b/\omega_b$ and $\Delta m_p/m_p$.

Now, we include the small scale information in the analysis to study how the damping tail is crucial for parameter determination from CMB in the third case. We make use of H_4 and H_5 , which is defined in the same way as H_2, H_3 , that is,

$$H_4 \equiv \left(\frac{\Delta T_{\ell_4}}{\Delta T_{\ell_1}} \right)^2, \quad H_5 \equiv \left(\frac{\Delta T_{\ell_5}}{\Delta T_{\ell_1}} \right)^2. \quad (38)$$

In other words, H_4 is the ratio of the fourth peak ℓ_4 to the first peak ℓ_1 height and H_5 is the ratio of the fifth peak ℓ_5 to the first peak ℓ_1 . Adding the relations between $\Delta H_4/H_4, \Delta H_5/H_5$

and the variations of the cosmological parameters, (34) becomes

$$\begin{pmatrix} \frac{\Delta \ell_1}{\ell_1} \\ \frac{\Delta H_1}{H_1} \\ \frac{\Delta H_2}{H_2} \\ \frac{\Delta H_3}{H_3} \\ \frac{\Delta H_4}{H_4} \\ \frac{\Delta H_5}{H_5} \end{pmatrix} = \begin{pmatrix} 0.069 & -0.109 & -0.206 & 0.162 & 0 & 0.087 \\ 0.414 & -0.408 & -0.294 & 2.666 & -0.093 & 0.127 \\ -0.647 & -0.045 & 0 & 1.183 & 0 & -0.783 \\ -0.415 & 0.438 & 0 & 0.760 & 0 & -0.567 \\ -0.478 & 0.315 & 0 & 1.608 & 0 & -0.868 \\ 0.665 & 0.493 & 0 & 1.828 & 0 & -0.794 \end{pmatrix} \begin{pmatrix} \frac{\Delta \omega_b}{\omega_b} \\ \frac{\Delta \omega_m}{\omega_m} \\ \frac{\Delta h}{h} \\ \frac{\Delta n_s}{n_s} \\ \frac{\Delta \tau}{\tau} \\ \frac{\Delta m_p}{m_p} \end{pmatrix}. \quad (39)$$

We can recast this equation through the relevant approximation in

$$\begin{pmatrix} \frac{\Delta \ell_1}{\ell_1} \\ \frac{\Delta H_1}{H_1} \\ \frac{\Delta H_2}{H_2} \\ \frac{\Delta H_3}{H_3} \\ \frac{\Delta H_4}{H_4} \\ \frac{\Delta H_5}{H_5} \end{pmatrix} \approx \begin{pmatrix} 0.1378 & -0.198 & 0.085 & -0.012 & -0.165 & 0.001 \\ 2.528 & -1.024 & 0.343 & -0.042 & 0.023 & -0.001 \\ 1.332 & 0.680 & 0.409 & 0.170 & -0.013 & -0.003 \\ 0.897 & 0.648 & -0.027 & -0.194 & -0.014 & -0.004 \\ 1.785 & 0.684 & 0.104 & -0.0469 & 0.012 & 0.006 \\ 2.007 & -0.046 & -0.789 & 0.069 & -0.013 & -0.001 \end{pmatrix} \begin{pmatrix} 0.960 \frac{\Delta n_s}{n_s} \\ -0.617 \frac{\Delta \omega_b}{\omega_b} - 0.658 \frac{\Delta m_p}{m_p} \\ -0.734 \frac{\Delta \omega_b}{\omega_b} - 0.579 \frac{\Delta \omega_m}{\omega_m} \\ -0.728 \frac{\Delta \omega_m}{\omega_m} - 0.582 \frac{\Delta m_p}{m_p} \\ 0.959 \frac{\Delta h}{h} \\ 0.994 \frac{\Delta \tau}{\tau} \end{pmatrix}. \quad (40)$$

All the elements of the last column in 6×6 matrix, which correspond to the coefficients of the parameter τ , are nearly zero, which is almost the same situation occurred in the previous case without $\Delta H_4/H_4$ and $\Delta H_5/H_5$. Unlike the previous case, there are no other (nearly) zero columns, which states that, as mentioned in section V, the strong degeneracy between ω_b and m_p can be resolved by observing not only ℓ_1, H_1, H_2, H_3 but also H_4, H_5 which possess the damping tail information.

Appendix B

In this appendix, we briefly report a forecast for constraints from the future CMB survey, in particular, the PLANCK survey [71]. The parameters for the instrumental design for PLANCK survey are shown in TABLE II and mock data are generated by the publicly available FuturCMB code [72]. The fiducial parameter values are the same as those adopted in section II and we perform the MCMC analysis in the same way as in section IV. Throughout this appendix, we assume the model between time variations of physical constants introduced in section III.

TABLE II: Assumed instrumental parameters for PLANCK survey. f_{sky} is the fraction of the observed sky, θ_{FWHM} is Gaussian beam width at FWHM, σ_T and σ_P are temperature and polarization noise, respectively.

f_{sky}	bands[GHz]	$\theta_{FWHM}[arcmin]$	$\sigma_T[\mu K]$	$\sigma_P[\mu K]$
0.65	100	9.5	6.8	10.9
	143	7.1	6.0	11.4
	217	5.0	13.1	26.7

TABLE III: MCMC results on the mean values, 68% and 95% confidence intervals of cosmological parameters.

	varying α, m_e and m_p
$100\Omega_B h^2$	$2.274^{+0.025+0.050}_{-0.025-0.049}$
$\Omega_{DM} h^2$	$0.111^{+0.002+0.003}_{-0.002-0.003}$
τ	$0.087^{+0.002+0.009}_{-0.003-0.009}$
n_s	$0.963^{+0.009+0.017}_{-0.009-0.018}$
$\log(10^{10} A_s)$	$3.065^{+0.011+0.023}_{-0.011-0.023}$
H_0	$71.9^{+1.0+2.1}_{-1.1-2.0}$
$\Delta\alpha/\alpha$	$0.0000021^{+0.0002+0.0003}_{-0.0002-0.0003}$
$\Delta\mu/\mu$	$0.0003^{+0.016+0.030}_{-0.015-0.031}$

In TABLE III, we show the forecast for the constraints on the cosmological parameters as the mean values, 68% and 95% confidence intervals. As is commented in the concluding section and appendix A, future CMB survey can constrain time variation of the physical constants more tightly and the limit on $\Delta\alpha/\alpha$ or $\Delta\mu/\mu$ from PLANCK experiment is expected to be about one order tighter than the constraint from the WMAP 5-year data and comparable to the constraint from BBN [31].

-
- [1] P. A. M. Dirac, *Nature* **139** (1937) 323.
 - [2] P. A. M. Dirac, *Proc. Roy. Soc. Lond. A* **165** (1938) 199.
 - [3] J. D. Bekenstein, *Phys. Rev. D* **25**, 1527 (1982).
 - [4] J. D. Barrow, H. B. Sandvik and J. Magueijo, *Phys. Rev. D* **65** (2002) 063504 [arXiv:astro-ph/0109414].
 - [5] J. D. Barrow and J. Magueijo, *Phys. Rev. D* **72** (2005) 043521 [arXiv:astro-ph/0503222].
 - [6] J. P. Uzan, *Rev. Mod. Phys.* **75** (2003) 403 [arXiv:hep-ph/0205340].
 - [7] T. Chiba, T. Kobayashi, M. Yamaguchi and J. Yokoyama, *Phys. Rev. D* **75** (2007) 043516 [arXiv:hep-ph/0610027].
 - [8] M. Dine and N. Seiberg, *Phys. Lett. B* **162** (1985) 299.
 - [9] R. Brustein and P. J. Steinhardt, *Phys. Lett. B* **302** (1993) 196 [arXiv:hep-th/9212049].
 - [10] P. P. Avelino, C. J. A. Martins, N. J. Nunes and K. A. Olive, *Phys. Rev. D* **74** (2006) 083508 [arXiv:astro-ph/0605690].
 - [11] P. P. Avelino, *Phys. Rev. D* **78** (2008) 043516 [arXiv:0804.3394 [astro-ph]].
 - [12] Y. Fujii, *Phys. Lett. B* **660** (2008) 87 [arXiv:0709.2211 [astro-ph]].
 - [13] Y. Fujii, *Phys. Lett. B* **671** (2009) 207 [arXiv:0809.0411 [astro-ph]].
 - [14] T. Dent, S. Stern and C. Wetterich, *JCAP* **0901** (2009) 038 [arXiv:0809.4628 [hep-ph]].
 - [15] P. P. Avelino, arXiv:0903.0617 [astro-ph.CO].
 - [16] J. K. Webb, V. V. Flambaum, C. W. Churchill, M. J. Drinkwater and J. D. Barrow, *Phys. Rev. Lett.* **82** (1999) 884 [arXiv:astro-ph/9803165].
 - [17] J. K. Webb *et al.*, *Phys. Rev. Lett.* **87** (2001) 091301 [arXiv:astro-ph/0012539].
 - [18] M. T. Murphy *et al.*, *Lecture Notes in Phys.*, **131** (2004), 648.
 - [19] M. T. Murphy, J. K. Webb and V. V. Flambaum, *Mon. Not. R. Astron. Soc.* **384** (2008), 1053, astro-ph/0612407.
 - [20] M. T. Murphy, J. K. Webb and V. V. Flambaum, *Phys. Rev. Lett.* **99** (2007), 239001, arXiv:0708.3677.
 - [21] P. Molaro, D. Reimers, I. I. Agafonova and S. A. Levshakov, *Eur. Phys. J. ST* **163** (2008) 173 [arXiv:0712.4380 [astro-ph]].
 - [22] H. Chand, R. Srianand, P. Petitjean and B. Aracil, *Astron. Astrophys.* **417** (2004) 853

- [arXiv:astro-ph/0401094].
- [23] R. Srianand, H. Chand, P. Petitjean and B. Aracil, Phys. Rev. Lett. **92** (2004) 121302 [arXiv:astro-ph/0402177].
 - [24] T. Chiba, In Proc. Frontier of Cosmology and Gravitation, (YITP, Kyoto, April 25-27, 2001) [arXiv:gr-qc/0110118].
 - [25] S. Blatt et al., Phys. Rev. Lett. **100** (2008), 140801, arXiv:0801.1874.
 - [26] T. M. Fortier *et al.*, Phys. Rev. Lett. **98** (2007) 070801.
 - [27] E. Peik, B. Lipphardt, H. Schnatz, T. Schneider, C. Tamm and S. G. Karshenboim, Phys. Rev. Lett. **93** (2004) 170801.
 - [28] M. Fischer et al., Phys. Rev. Lett. **92** (2004), 230802.
 - [29] T. Rosenband et al., Science **319** (2008), 1808.
 - [30] Y. Fujii *et al.*, In Proc. Int. Conf. on Nuclear Data for Science and Technology, 2001 [arXiv:hep-ph/0205206].
 - [31] K. Ichikawa and M. Kawasaki, Phys. Rev. D **65** (2002) 123511 [arXiv:hep-ph/0203006].
 - [32] K. Ichikawa, T. Kanzaki and M. Kawasaki, Phys. Rev. D **74** (2006) 023515 [arXiv:astro-ph/0602577].
 - [33] G. Rocha, R. Trotta, C. J. A. Martins, A. Melchiorri, P. P. Avelino, R. Bean and P. T. P. Viana, Mon. Not. Roy. Astron. Soc. **352** (2004) 20 [arXiv:astro-ph/0309211].
 - [34] P. Stefanescu, New Astron. **12** (2007) 635 [arXiv:0707.0190 [astro-ph]].
 - [35] M. Nakashima, R. Nagata and J. Yokoyama, Prog. Theor. Phys. **120**, 1207 (2008) [arXiv:0810.1098 [astro-ph]].
 - [36] E. Menegoni, S. Galli, J. Bartlett, C. J. A. Martins and A. Melchiorri, arXiv:0909.3584 [astro-ph.CO].
 - [37] E. Reinhold, R. Buning, U. Hollenstein, A. Ivanchik, P. Petitjean and W. Ubachs, Phys. Rev. Lett. **96** (2006) 151101.
 - [38] V. V. Flambaum, M. G. Kozlov and M. G. Kozlov, Phys. Rev. Lett. **98** (2007) 240801 [arXiv:0704.2301 [astro-ph]].
 - [39] M. T. Murphy, V. V. Flambaum, S. Muller and C. Henkel, Science **320** (2008) 1611 [arXiv:0806.3081 [astro-ph]].
 - [40] B. A. Campbell and K. A. Olive, Phys. Lett. B **345**, 429 (1995) [arXiv:hep-ph/9411272].
 - [41] P. Langacker, G. Segre and M. J. Strassler, Phys. Lett. B **528**, 121 (2002)

- [arXiv:hep-ph/0112233].
- [42] T. Dent and M. Fairbairn, Nucl. Phys. B **653** (2003) 256 [arXiv:hep-ph/0112279].
 - [43] C. M. Muller, G. Schafer and C. Wetterich, Phys. Rev. D **70** (2004) 083504 [arXiv:astro-ph/0405373].
 - [44] T. Dent, S. Stern and C. Wetterich, Phys. Rev. D **78** (2008) 103518 [arXiv:0808.0702 [hep-ph]].
 - [45] X. Calmet and H. Fritzsch, Eur. Phys. J. C **24** (2002) 639 [arXiv:hep-ph/0112110].
 - [46] X. Calmet and H. Fritzsch, Phys. Lett. B **540** (2002) 173 [arXiv:hep-ph/0204258].
 - [47] K. A. Olive, M. Pospelov, Y. Z. Qian, A. Coc, M. Casse and E. Vangioni-Flam, Phys. Rev. D **66** (2002) 045022 [arXiv:hep-ph/0205269].
 - [48] S. J. Landau, M. E. Mosquera, C. G. Scoccola and H. Vucetich, Phys. Rev. D **78** (2008) 083527 [arXiv:0809.2033 [astro-ph]].
 - [49] C. G. Scoccola, S. J. Landau and H. Vucetich, Phys. Lett. B **669** (2008) 212 [arXiv:0809.5028 [astro-ph]].
 - [50] M. Dine, Y. Nir, G. Raz and T. Volansky, Phys. Rev. D **67** (2003) 015009 [arXiv:hep-ph/0209134].
 - [51] S. Hannestad, Phys. Rev. D **60** (1999) 023515 [arXiv:astro-ph/9810102].
 - [52] M. Kaplinghat, R. J. Scherrer and M. S. Turner, Phys. Rev. D **60** (1999) 023516 [arXiv:astro-ph/9810133].
 - [53] J. Kujat and R. J. Scherrer, Phys. Rev. D **62** (2000) 023510 [arXiv:astro-ph/9912174].
 - [54] J. J. Yoo and R. J. Scherrer, Phys. Rev. D **67** (2003) 043517 [arXiv:astro-ph/0211545].
 - [55] <http://camb.info/>
 - [56] J. Dunkley *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **180** (2009) 306 [arXiv:0803.0586 [astro-ph]].
 - [57] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **180** (2009) 330 [arXiv:0803.0547 [astro-ph]].
 - [58] W. Hu and N. Sugiyama, Astrophys. J. **471** (1996) 542 [arXiv:astro-ph/9510117].
 - [59] W. Hu, M. Fukugita, M. Zaldarriaga and M. Tegmark, Astrophys. J. **549**, 669 (2001) [arXiv:astro-ph/0006436].
 - [60] C. Lovelace, Phys. Lett. B **135** (1984) 75.
 - [61] E. S. Fradkin and A. A. Tseytlin, Phys. Lett. B **158** (1985) 316.
 - [62] C. G. Callan, E. J. Martinec, M. J. Perry and D. Friedan, Nucl. Phys. B **262** (1985) 593.

- [63] A. Sen, Phys. Rev. D **32** (1985) 2102.
- [64] A. Sen, Phys. Rev. Lett. **55** (1985) 1846.
- [65] C. G. . Callan, I. R. Klebanov and M. J. Perry, Nucl. Phys. B **278** (1986) 78.
- [66] M. R. Nolte *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **180** (2009) 296
[arXiv:0803.0593 [astro-ph]].
- [67] A. Lewis, A. Challinor and A. Lasenby, Astrophys. J. **538** (2000) 473 [arXiv:astro-ph/9911177].
- [68] A. Lewis and S. Bridle, Phys. Rev. D **66** (2002) 103511 [arXiv:astro-ph/0205436].
- [69] <http://cosmologist.info/cosmomc/>
- [70] W. L. Freedman *et al.* [HST Collaboration], Astrophys. J. **553** (2001) 47
[arXiv:astro-ph/0012376].
- [71] [Planck Collaboration], arXiv:astro-ph/0604069.
- [72] L. Perotto, J. Lesgourgues, S. Hannestad, H. Tu and Y. Y. Y. Wong, JCAP **0610** (2006) 013
[arXiv:astro-ph/0606227].